

Research Article

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Jana Daňková*, Pavel Mec, and Tereza Majstríková

Stiffness analysis of glued connection of the timber-concrete structure

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Abstract: This paper presents results of experimental and mathematical analysis of stiffness characteristics of a composite timber-concrete structure. The composite timber-concrete structure presented herein is non-typical compared to similar types of building structures. The interaction between the timber and concrete part of the composite cross-section is not based on metal connecting elements, but it is ensured by a glued-in perforated mesh made of plywood. The paper presents results of experimental and mathematical analysis for material alternatives of the solution of the glued joint. The slip modulus values were determined experimentally. Data obtained from the experiment evaluated by means of regression analysis. Test results were also used as input data for the compilation of a 3D model of a composite structure by means of the 3D finite element model. On the basis of result evaluation, it can be stated that the stress-deformation behaviour at shear loading of this specific timber-concrete composite structure can be affected by the type of glue used. Parameters of the 3D model of both alternative of the structure represent well the behaviour of the composite structure and the model can be used for predicting design parameters of a building structure.

Keywords: timber-concrete structure, shear test, glued in mesh, numerical modelling

1 Introduction

Timber-concrete and steel-concrete composite structures are currently used in structurally-engineered floor structures and bridge construction [1]. The principle consists in connecting a reinforced concrete slab, which is mainly subjected to compression in the structure, and a tim-

ber beam, which is predominantly subjected to bending. The connection of both parts of the cross-section occurs through the use of shear connectors. Shear connectors are defined by their material properties, geometrical shape and location within the composite cross-section; they form the shear connection system. This system ensures the redistribution of internal forces between different parts of the composite cross-section. Individual shear connectors are mainly exposed to shear stress. This kind of structure arrangement allows for much greater load-carrying capacity and stiffness of the structure, while the volume of material used can be significantly reduced [2]. Other advantages of timber-concrete composite structures include improved thermal-technical, acoustic [1] and fire properties [3] compared to traditional wooden structures.

According to the nature of load-slip relationships, timber-concrete composite structures can be divided into structures with rigid, semi-rigid and ductility connection systems [4]. The structures with ductility connection systems mainly include structures where the shear connectors are pin-type discrete metal mechanical fasteners such as nails, screws, staples or pins. Rigid connection is ensured through the use of glued-in shear connectors, continuous wood meshes [5], steel lattices glued to timber or flat glued joints. A special way of connection is provided by techniques where the connection is ensured by indentations in timber, interlayer profiling (formwork), or combinations of different types of connection such as grooved holes and dowels [6].

The load-slip relationship of the clamping system is a factor that greatly affects the efficiency of the shear connection [7]. One evaluation criterion for assessing the load-slip relationship of the shear connection system is the value of SLIP MODULUS [N/mm]. This value can be determined experimentally based on a shear test according to EN 26891 [8] or using table values [9]. A rigid connection is characterized by high slip modulus values and low finite slip values. A ductility connection is characterized by low slip modulus values and a generally higher finite slip. Both types of connection systems have their advantages and disadvantages. Rigid connection systems may disadvantageous when used in structures with large spans and doubts may arise in the design of such structures for long-

*Corresponding Author: Jana Daňková: VŠB - Technical University of Ostrava, Ostrava, Czech Republic, E-mail: jana.dankova@vsb.cz
Pavel Mec, Tereza Majstríková: VŠB - Technical University of Ostrava, Ostrava, Czech Republic

term loads and accidental events. Timber-concrete structures with a ductility connection do not provide significantly increased load-carrying capacity or overall structure stiffness.

The currently available theoretical and experimental research of timber-concrete composite structures with glued joints consider the glued joints to be a permanent and rigid connection without any slip. Therefore, under this assumption, glued joints have no inherent load-slip relationships. However, it is known that a glued joint itself is a structure that has certain strength, stiffness and durability qualities and affects the behaviour of the composite structure of which it is a part [10, 11].

Timber-concrete structures, which are the subject of the research presented, are patent-protected and were developed by the authors of this article. Earlier research, conducted by authors of this article in 2009-2011 [12], already showed that the properties of timber-concrete structures could be negatively influenced by the behaviour of the glued joint. The subsequent research, which took place in 2012-2015, was aimed at the possibility of controlling the properties of timber-concrete structures with glued-in perforated mesh.

The aim of this study was to verify the load-slip behaviour of a specific prototypical timber-concrete composite structure with a variable material solution to a glued joint. The results of this study demonstrated that the load-slip behaviour of this timber-concrete structure can be deliberately influenced by the selection of the glued joint material.

2 Materials and methods

2.1 Adhesives and glued joints

The research surrounding technologies and materials for glued joints is currently gaining much attention. Compared to joints with mechanical steel connectors, glued joints are simpler, stronger and more economical. To achieve a reliable glued joint, it is necessary that the adhesive strength of the glued joint be greater than the cohesive strength. To conduct the experiment, we therefore selected two types of adhesives whose declared specifications create good conditions for the formation of such a joint within the composite structure. The following adhesives were selected: polyurethane adhesive 1C (hereinafter referred to as 1C PUR) and an elastomer hybrid adhesive, a silane-modified polyether (hereinafter referred to as SMP = Silyl Modified Polymer), in a two-component adhesive

variant (hereinafter referred to as 2C SMP). Both adhesives cure due to atmospheric humidity. Both adhesives are intended for the formation of flexible glued joints with good adhesion to substrates. Both of them are certified commercial products supplied on the European market.

SMP adhesives are a newer technology. These adhesives combine the strength of polyurethanes and the durability of silicones [13], and are more environmentally friendly. Currently, these adhesives are used for applications in the aerospace and marine industries, in the manufacture and repair of railway wagons, etc. Generally, these materials are suitable for joining metallic materials, glass, some plastics, etc. Applications for wood and wood-based materials are not too common. The assembly of the samples was carried out using TEROSON MS 9399, a commercial product manufactured by Henkel AG & Co. KGaA, Germany.

Polyurethane adhesives are known as wood adhesives. Sometimes, they are also referred to as isocyanate adhesives. Polyurethane adhesives feature very good mechanical properties, high resistance to dynamic loads, high elasticity and very good resistance to weathering. Samples were produced using PURBOND[®], a commercial product manufactured by Henkel AG & Co. KGaA, Germany (held by Collano AG, Switzerland). It is a 1C PUR adhesive designed for the production of glued laminated timber, the creation of structural glued joints in timber structures, such as adhesive bonding of I-profile flanges, large finger joints, etc. The product meets the standard requirements of European legislation.

2.2 Timber-concrete structures with non-metallic glued-in mesh

The subject of the research was a timber-concrete structure designed for installation in floor structures in multi-story timber buildings. This construction is useful where there are requirements for higher utility loads. The cross-section of composite structures consists of a wooden part and reinforced-concrete section (see Fig. 1).

The wooden part of the composite cross-section is constructed from glued laminated timber and the cross-sectional dimension is defined in terms design requirements. In its upper part, there is a milled continuous groove intended for gluing a connection mesh. The connection mesh is made out of 8-mm thick surface-untreated waterproof plywood (see Tab. 1). Its geometry is shown in Fig. 2. In its upper part, the mesh is equipped with circular openings through which the reinforcement is pulled. Through this arrangement, the mesh is analogous

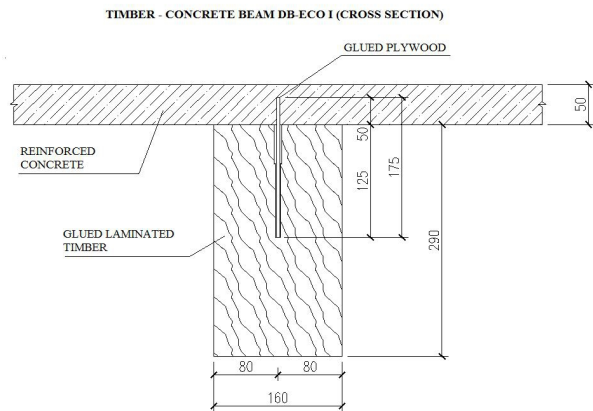


Figure 1: The timber-concrete structure with non-metallic glued-in mesh. Cross-sectional geometry.

to steel connection meshes as known in steel-concrete structures [14, 15]. In its bottom part, the mesh is fitted with design modifications that ensure an even distribution of glue over the plywood's surface. The mesh is glued into the prepared line gap in the upper face of the wooden part of the profile whereas, after being glued in, the upper part of the mesh with circular openings extends past the upper face of the wooden profile by 50 mm (see Fig. 3).

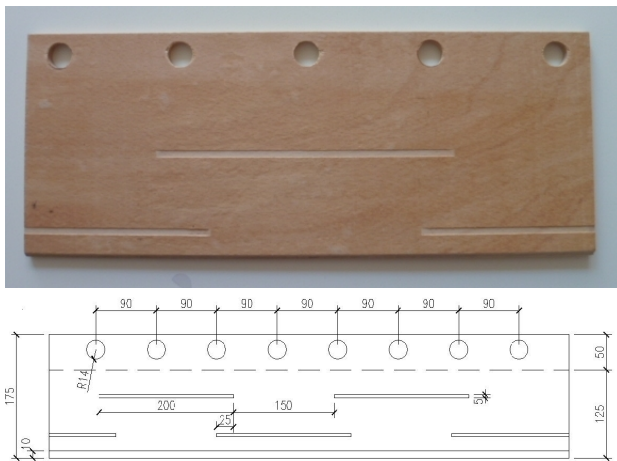


Figure 2: Connection mesh: Photo and the design documentation.

Production of samples

A total of 6 samples in two sets were produced to conduct the experiment. The geometric arrangement of samples was unilateral [4]. The cross-sectional dimensions of the wooden part of the profile were 160/300 mm; the concrete slab had a thickness of 60 mm and outer dimensions of

500/500 mm. The length of the bond line gap was 397 mm. B 500B reinforcement was used in 10 pieces of reinforcing bars for each sample; the reinforcing bars' diameter was 12 mm. Shear connection reinforcement was pulled through the circular openings and secondary reinforcement was added to ensure the geometry of the concrete sample (see Fig. 4, 5). Concreting followed afterwards.



Figure 3: Production of samples. The wooden part of the profile with the encompassed mesh.

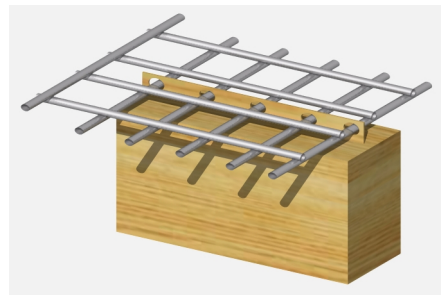


Figure 4: Production of samples. Completion of reinforcement.



Figure 5: Production of samples. Concreting and assembly of samples.

Table 1: Characteristics of the materials used.

		Properties
Wooden part of the composite cross-section	Density [kg/m^3]	350
	E_L [Mpa]	13 000
	E_T [MPa]	620
	E_N [Mpa]	620
	G_{LT} [MPa]	700
	G_{LN} [MPa]	700
	G_{TN} [MPa]	40
	Density [kg/m^3]	350
Connection mesh	E_L [Mpa]	10 000
	E_T [MPa]	7 000
	E_N [Mpa]	2 000
	G_{LT} [MPa]	200
	G_{LN} [MPa]	200
	G_{TN} [MPa]	2 000
Teroson MS 9399	E [MPa]	3
	Shear strength [MPa]	2
Reinforcement	Tension strength [Mpa]	500
	Bulk density [kg/m^3]	2 310
Concrete	Compression strength (cube) [Mpa]	52.5
	E [Gpa]	20 000

2.3 Shear tests

The experimental determination of the stiffness characteristics, the SLIP MODULUS [N/mm], in the new type of shear connection followed the procedure according to EN 26891 Timber structures–Joints made with mechanical fasteners–General principles for determining strength and slip characteristics [8]. This standard specifies the test procedure, requirements for the test equipment, requirements for the identification and interpretation of data recorded, etc. The standard, however, does not specify the size or shape of the samples or details regarding the arrangement of samples in the test device. The arrangement of the sample in the test device was analogous to published experiments [11, 16].

Prior to the test, each test specimen was air conditioned for at least 7 days in an environment with the following parameters: $t=20^\circ\text{C}+3^\circ\text{C}$ and $\text{R.H.} = 50+5\%$. Wood moisture content before the beginning of the test was measured by an electrical resistance moisture meter and measured moisture values were recorded. Wood moisture content ranged between 10–12%.

Each sample was placed in the test device so that the load acted evenly on the entire cross-sectional area of the specimen's wooden part. The test specimen was secured against displacement and buckling; slip sensors with a

measurement accuracy of 0.01 mm were located on both sides of the contact gap between the wooden and concrete portions of the sample, approximately in the middle of the length of the gap. The slip reading provided a fully digitized output. The loading speed was constant, 0.1 kN/s, and met the requirements of EN 26891.

Test procedure:

- maximum load F_{est} was estimated
- samples were loaded at the level of $0.4 F_{est}$ – step 1
- the load was maintained at this value for 30 seconds – step 2
- the load was continuously reduced to the level of $0.1 F_{est}$ – step 3
- the load was maintained at the level of $0.1 F_{est}$ for 30 seconds – step 4
- the load was continuously increased until the sample failed, slips were recorded at every 10% increment in the load force – step 5

Data were analysed using MATLAB[®] software.

Table 2: Results of the testing.

Sample	Maximal shear force	Maximal slip
	F_{max} [kN]	[mm]
PUR-1	48.14	5.82
PUR-2	54.15	5.41
PUR-3	53.38	6.47
SMP-1	34.89	21.25
SMP-2	37.45	27.21
SMP-3	26.89	17.09

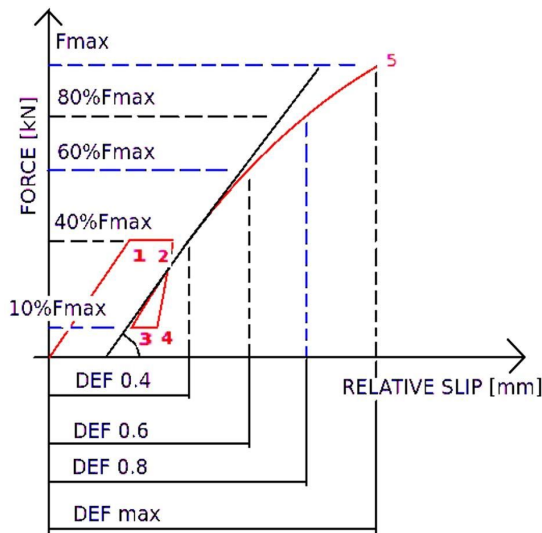


Figure 6: Method of loading samples and evaluation of data.

3 Results and their analysis

3.1 Results of the experiment

The set of samples assembled using the PURBOND® adhesive (sample PUR-1,2,3) showed generally higher maximal shear force F_{max} values at the sample's failure together with minor finite slip (see Tab. 2). The set of samples assembled using the TEROSON MS 9399 adhesive (sample SMP-1,2,3) showed generally lower maximal shear force F_{max} values, but the final slip capacity was much higher (see Tab. 2). Values of the SLIP MODULUS $K_{0.4}$, $K_{0.6}$ and $K_{0.8}$ were determined (see Tab. 3). To determine the stiffness characteristics, we used the measured slip value (see Fig. 6), not the modified slip value as given by the standard [8]. Since it was the first experiment on this type of structure, the requirement for a good estimate of F_{est} was not achieved. The estimated maximal force F_{est} differed from the actual final F_{max} by more than 20%. The recorded load – maximal slip curves were evaluated (see Fig. 7, 8).

The shear force – relative slip curve represents the course of a function that expresses the relationship between the stress and average slip. Average slip is an absolute value of the average between the right and left side slip. Data were obtained from the slip sensors located on both sides of the contact gap between the concrete and wood.

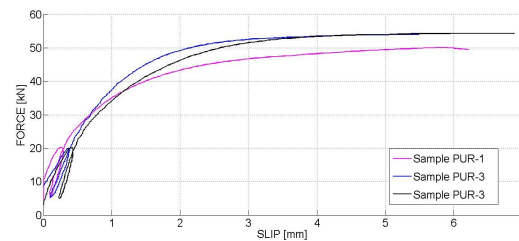


Figure 7: Shear test results for sample PUR-1, 2, 3. Shear force – relative slip curves.

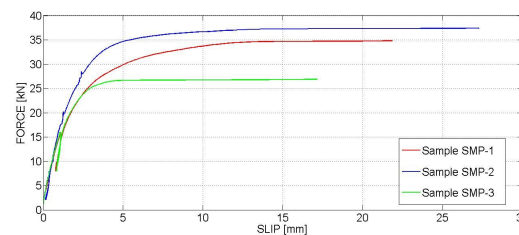


Figure 8: Shear test results for sample SMP-1, 2, 3. Shear force – relative slip curves.

Table 3: Evaluation of stiffness characteristics.

Sample	$K_{0,4}$ [N/mm]	$K_{0,6}$ [N/mm]	$K_{0,8}$ [N/mm]	Description of the sample failure
PUR-1	65 574	43 869	26 893	Mesh destruction
PUR-2	53 555	43 033	32 571	Mesh destruction
PUR-3	44 367	35 694	25 582	Mesh destruction
average	54 499	40 865	28 349	
minimum value	44 367	35 694	25 582	
maximum value	65 574	43 869	32 571	
percentile 5	45 286	36 428	25 713	
σ	10 635	4 498	3 715	
SMP-1	12 857	11 063	7 259	Cohesive failure of the glued joint
SMP-2	16 927	13 695	10 331	Cohesive failure of the glued joint
SMP-3	11 548	13 230	10 969	Cohesive failure of the glued joint
average	13 777	12 663	9 520	
minimum value	11 548	11 063	7 259	
maximum value	16 927	13 695	10 969	
percentil 5	11 679	11 280	7 485	
σ	2 805	1 405	1 984	

3.2 A regression analysis of the experiment's data

The nature of the load-slip functional dependence provides information for developing a design model. If this dependence can be easily expressed as a linear function, it is possible to say that there are good prospects for the development of the design model. The part of the curve between load steps 4 and 5 (see Fig. 6) was evaluated using the MATLAB® software tool. This software tool was used for performing a regression analysis curve of the shear force – relative slip for each sample in each section of the curve between 0.1 and 0.6 F_{max} . The results of the analysis are shown in Table 4.

3.3 Numerical modelling

A 3D finite element model of the tested sample was prepared (see Fig. 9) in Salome-Meca software [22]. The model was made in three parts in the case of the PU sample and four parts in the case of the MS-polymer sample. There was no considered friction between the timber and concrete part because the experimentally-tested samples did not have any friction.

For concrete, the simple elastic model was used. This was because the concrete part of a composite system is tougher than other wooden parts. The elastic moduli was measured on the cylindrical sample. The Poisson ratio was

assumed at 0.2. For the purposes of modelling, the reinforcement was not considered due to the high stiffness of the concrete part. Tension in the concrete part was not so great; therefore, the reinforcement did not play an important role.

The material properties of the timber part (timber beam) were not determined by any test. It was modelled as an orthotropic material. The material properties (see Tab. 1) in the three orthogonal axes were used according to the literature resources [17]. The timber was considered to be a linear elastic orthotropic material.

The material properties of the plywood connector were the most important aspect for the numerical model because the plywood is the part that is mainly responsible for determining the toughness of a composite system. Plywood is made from five-ply wood bonded with adhesive. Every ply is laid by its longitudinal axis perpendicular to longitudinal axis of the other ply. Three plies of plywood have their longitudinal axis parallel to the direction of the force and two plies are perpendicular.

However, plywood does not have the same number of plies in both axes and in the model assumes the same properties of plywood in both plane axes. The material properties of the plywood were obtained from literature resources [18].

Material characteristics for modelling adhesive joint TEROSON MS 9399 were taken from the technical documentation for the product [19]. The material was modelled

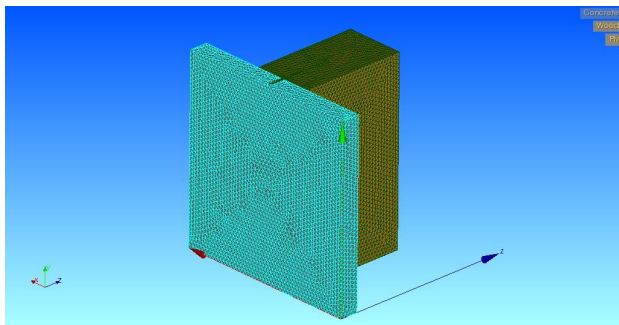
Table 4: Regression analysis of selected parts of curves shear force – relative slip.

Sample	Linear model	Coefficients of the linear model		Goodness of the fit		
		p1	p2	SSE	RMSE	R-square
PUR-1	$f(x) = (p1 \cdot x) + p2$	46.02	4.309	756.6	1.776	0.935
PUR-2	$f(x) = (p1 \cdot x) + p2$	41.41	2.612	275.3	0.995	0.985
PUR-3	$f(x) = (p1 \cdot x) + p2$	42.56	-1.434	879.5	1.795	0.949
SMP-1	$f(x) = (p1 \cdot x) + p2$	11.98	0.163	219.2	0.802	0.955
SMP-2	$f(x) = (p1 \cdot x) + p2$	14.31	1.220	562.5	1.023	0.970
SMP-3	$f(x) = (p1 \cdot x) + p2$	21.55	-9.459	14.67	0.259	0.988

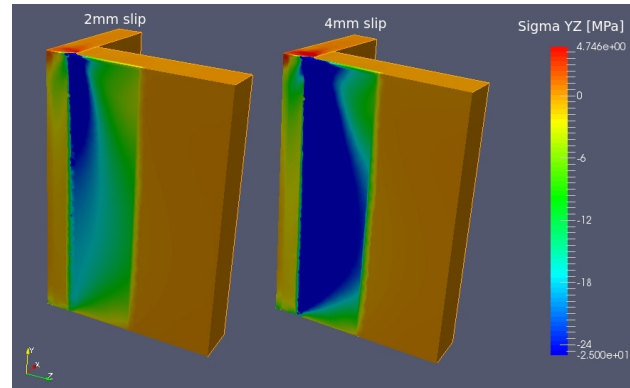
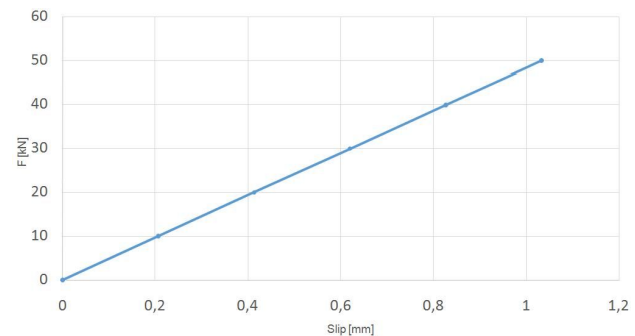
in 2 mm thick with the assumption limit breach of at 2 MPa shear strength, as indicated by the producer.

In the timber-concrete composite with the SMP adhesive, it was necessary to use material properties because the adhesive is quite thin (about 1-2 mm) and it is modelled as a plastic material. From the tests, it was obvious that toughness of this system was defined by the mechanical parameters of the SMP adhesive.

Based on observations during the test can be confirmed, that the behaviour of the composite system for a given loading conditions (shear), is determined by the mechanical properties of the adhesive.

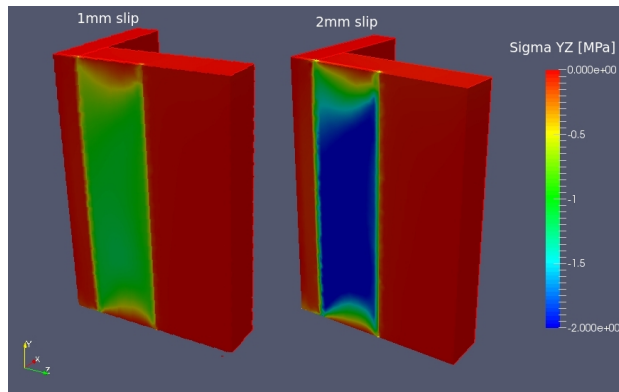
**Figure 9:** Model of the sample. The orientation with respect to coordinate axes.

The results of numerical modelling are consistent with realized experiments. The samples of PUR is exceeded on shear strength of the plywood in cross-section of the mesh. The adhesive bonding is the weakest element of a composite system for the samples of SMP. The destruction of the sample will occur exceeding the shear strength of the bond (see figure 10, 11). The results of numerical modelling confirm that stress in concrete and glued laminated timber is unimportant.

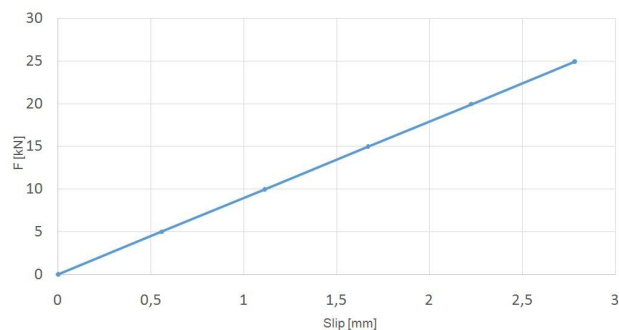
**(a)****(b)****Figure 10:** Model of the sample PUR. Shear stress on the plane YZ (a) and obtained linear curve shear force – relative slip (b).

4 Discussion and conclusion

The timber-concrete composite structure was verified in relation to the stress-deformation behaviour at the shear load in two glued joint material alternatives. For both types of samples, the regression analysis proved linear dependence between force and deformation up to the level of 60% of the maximum load of F_{max} . Results of the experiments prove that stress-deformation behaviour of the composite structure is obviously influenced by the adhesive material. Samples glued with 2C SMP TEROSON MS 9399



(a)



(b)

Figure 11: Model of the sample SMP. Shear stress on the plane YZ (a) and obtained linear curve shear force – relative slip (b).

adhesive showed lower values of strength and stiffness characteristics. However, their high final deformation was very high and exceeded the limit values of 15 mm with all sampled. The high ability of deformation and low standard deviation values (range average σ) for the slip modulus values - this behaviour of a composite timber-concrete structure is extraordinary among all the currently known systems [20, 21].

Samples glued with a 1C PUR PURBOND® adhesive showed higher strength, higher slip modulus values and lower final deformations than the samples glued with the 2C SMP TEROSON MS 9399 adhesive. However, the maximum slip values, which achieved approximately 6 mm, are extraordinary compared to other glued timber-concrete composite systems. Numerical models of both types of samples took into account the importance of the glued joint, especially for samples glued with the 2C SMP TEROSON MS 9399 adhesive. The behaviour of models and real samples was identical during the experiment.

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Nomenclature

E	Young's modulus
E_L, E_T, E_N	Young's modulus in longitudinal (L), transverse (T) and normal (N) plane
F_{max}	Maximal force (load)
F_{est}	Estimated maximal force
G_{LT}, G_{LN}, G_{TN}	Shear modulus in longitudinal-transverse (LN), transverse-normal (TN) and longitudinal-normal (N) plane
$K_{0.4}$	Slip moduli for 40% maximal force F_{max}
$K_{0.6}$	Slip moduli for 60% maximal force F_{max}
$K_{0.8}$	Slip moduli for 80% maximal force F_{max}
N, kN	Newton, Kilonewton – unit of force
Pa, MPa, GPa	Pascal, Megapascal, Gigapascal – unit of ultimate tensile strength
R.H. [%]	Relative humidity
RMSE	Root mean square error
R-square	Correlation coefficient
SSE	Sum of square errors
1C	adhesive One-component adhesive
2C	adhesive Two-component adhesive
3D	model Tree-dimensional model
kg	Kilogram – unit of mass
mm	Millimetre – unit of length
m^3	Cubic metre – unit of volume
t [°C]	Temperature
s	Second – unit of time
σ	Standard deviation

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